MODELLING STOCK MARKET DEPENDENCE BETWEEN NIGERIA'S AND OTHER STOCK MARKETS USING UNCONDITIONAL COPULA

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ABSTRACT

In this paper, we use copula approach to examine Nigerian stock market's dependence structures with the markets of South Africa, Egypt, Argentina, Indonesia, Mexico, Turkey, Hong Kong, United Kingdom and 2 United States indices. The stock indices were selected as the largest economies in Africa, the top performing indices of 2017 as well as to understand the relationship between developed and developing markets. We examine the marginals using ARMA GJR-GARCH with Normal and t error distributions, while Gumbel, Clayton, Rotated Gumbel, Rotated Clayton, Frank, student t and symmetrized Joe–Clayton copulas with Gaussian as the benchmark are employed to analyze the joint distributions. To help us select the best copula, we perform a range of goodness-of-fit tests on daily returns from 4th January 2007 to 29th December, 2017 of these markets. Our study is limited to pairwise dependence between Nigerian against each of the indices. The Student t copula was found to fit the data very well and emerged as the model to describe pair wise dependence between Nigerian stock index against nine other world's stock markets. The study also revealed that the Indonesian stock market is the most integrated index with Nigerian stock market. The findings of the study have important implications for portfolio analysis by market players, the choice of which stock market the NSE selects for a deepened relationship as well as for contagion analysis.

Keywords: copula, dependence, correlation, stock market, tail dependence.

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1.0 Introduction

Modern risk management and portfolio allocation requires reliable modelling of dependence among markets and risky assets. The dependence structure has practical implications that include asset pricing, risk management, hedging, contagion, portfolio allocation, international diversification and market integration. Using the wrong model for dependence between assets or markets can result in incorrect portfolio diversification choices and may intensify systemic risk. Furthermore, there are financial stability implications of dependence between markets and systemically important firms (Clare and Lekkos, 2000).

Numerous empirical studies have considered the dependence between stock markets. Longin and Solnik (2001) and Ang and Chen (2002) study dependence between international and domestic markets using linear correlations. Apart from multivariate normal distribution using linear correlation coefficient, copula has emerged as a very popular method for modelling dependence among assets and markets. Copulas are functions that connect multivariate distributions to their one-dimensional margins (Sklar, 1959). Copula has extensive applications in finance: Rodriguez (2007) measure dependence between equity markets with copula; Dias and Embrechts (2010) and Patton (2006) study foreign exchange markets using copula, amongst several others. Extreme Value Theory (EVT) as the marginal and copula have been extensively employed to investigate dependence in the presence of non-normality in data. Clemente and Romani (2003) use EVT and copula to model operational risk using insurance data. Hotta et al. (2008) use copula to study market risk for a portfolio consisting of Nasdaq and S&P 500 indices. Hsu et al. (2012) incorporate a combination of copula and EVT to access portfolio risk in six Asian markets.

Mensah and Alagidede (2016) examine the dependence structure between two developed and four emerging African stock markets using copula. The study used FTSE/JSE All Share for South Africa, Hermes Financial for Egypt, Nigeria All Share Index, Nairobi SE for Kenya, FTSE 100 for United Kingdom and the S&P 500 for the United States. They find weak dependence between African and developed stock markets, evidence of asymmetry and observe weak tail dependence among all the countries.

Ogunyiola, Njenga and Mwita (2016) estimate the dependence structure between international stock markets using Clayton copula. Their results reveal maximum possible loss of market value is 75.9% and 77.6% with a confidence interval of 90% for the Kenya-Nigeria and Kenya-South Africa portfolios, respectively, during the crisis period (2007-2009).

This paper extends the study of linkages and relationships between the Nigeria's and the other stock markets in three ways. First, we use a much bigger sample than prior studies in order to capture several events that had significant impact in Nigerian equity market like the 2007 global crisis and the 2009 national banking and equity crisis. Second, we investigate the dependence between the Nigeria and nine other equity markets using a wide variety of different copula models than previously studied which incorporate asymmetry and flexible tail behaviour. Third, we show how using an unconditional copula model can help in portfolio diversification and improve active asset allocation for investors interested in Nigerian markets. The NSE or capital market regulators can also use the result of the study to decide which equity market it should consider for a closer relationship.

Nigeria is the largest economy in Africa, followed by South Africa and Egypt as the second and third biggest economies, respectively. We are therefore interested in the dependence between the stock markets of these countries due to their positions in the African continent. Also, the best performing stock markets of 2017[1] were the US Dow Jones industrial average that shot up by 25%, the US S&P 500 surged by 19% and Hong Kong Hang Seng charged ahead by 36%. Other top performers were Turkey's benchmark index rallied by 48%, Argentina Merval gained 77% and the Nigerian NSE All-Share Index recorded 42% in 2017. We are also interested in the dependence between these countries' stock indices and Nigeria's as the top performing stock indices in 2017.

In this paper, we use copulas to analyse the relationship between Nigeria stock exchange and the markets of South Africa, Egypt, Argentina, Indonesia, Mexico, Turkey, Hong Kong, United Kingdom and 2 United States indices. We model the log-returns on each stock market individually, and we account for the dependence between them by copula functions. We perform a range of goodness-of-fit tests to help us select the best copula as a measure of dependence between Nigeria and each of the selected stock markets.

The reminder of the paper is structured as follows: Section 2 provides a brief review of the empirical and theoretical literature on dependence modelling using copula. Section 3 discusses methodology including the data and copula model specifications. Section 4 presents the empirical results, including the estimation of marginal and copula models. Section 5 concludes the paper.

2.0 Review of empirical and theoretical literature

There are two components of a multivariate model: the univariate or marginal which describes each of the variables and the dependence structure between these marginal variables. *Since the 1960s, Mandelbrot (1963) and Fama (1965)* observe that univariate asset return distributions are not normally distributed as evidenced by excess kurtosis (or fat tails) and skewness beyond normal distribution.

Numerous empirical studies have examined the dependence between stock markets. For instance, using linear correlation, Longin and Solnik (2001) and Ang and Chen (2002) show that correlation between stock market returns is not constant over time in both international and domestic markets.

In most early studies, the linear correlation coefficient was used as a measure of dependence in stock markets and in general financial econometrics. Presently, it has been well-established that linear correlation is a natural dependence measure for only spherical and elliptical distributions which include multivariate normal but distributions of the real world are seldom in this class (Embrechts, McNeil and Straumann, 2002). Linear correlation is invariant under strictly increasing linear transformations, which is a desirable

[1] http://money.cnn.com/2017/12/27/investing/best-stock-markets-2017-world/index.html

feature. However, it is not invariant under nonlinear strictly increasing transformations and therefore linear correlation is not a measure of concordance (Embrechts, Lindskog and McNeil, 2003).

Jondeau and Rockinger (2003) examine the behavior of some stock market returns based on heavy-tailed distributions and show that linear correlation is not an ideal dependency measure. Thus, assuming multivariate normality or using linear correlation coefficient for measuring stock market dependence can lead to inaccurate hedging choice, wrong portfolio decision or underestimate portfolio risk.

In addition to being non-normally distributed, stock returns have expressed more dependence in the lower tail when they are extremely negative than in the upper tail when they are extremely positive. This is a phenomenon that linear correlation cannot capture (Longin and Solnik, 2001; Ang and Chen, 2002). Poon et al. (2004) report asset returns asymmetric dependence, which is a rejection of multivariate normality. Asymmetric dependence is where returns exhibit greater correlation during market crashes than market booms as shown by Patton (2006) using copula functions.

Kendall's tau and Spearman's rho as copula measures, are the best substitutes of linear correlation coefficient (Embrechts et al., 2003). A desirable feature of Kendall's tau and Spearman's rho is that they are invariant under strictly increasing component-wise transformations. Another desirable feature is that for continuous random variables all values in the interval $[-1;1]$ can be obtained for Kendall's tau or Spearman's rho by a suitable choice of the underlying copula but that is however not the case with linear correlation.

McNeil, Frey and Embrechts, (2005) and Embrechts et al. (2002) suggest copula and rank correlations for modelling dependence concepts. McNeil et al. (2005) describe why copula emerged as a very versatile in risk management for several reasons. First, extreme dependence between assets can be described by copula, whether the dependence is assumed constant or time-varying. Second, the copula approach enables several marginal models to be joined or 'coupled' with a variety of possible dependence specifications. Finally, tail dependence, both symmetric and asymmetric dependence as well as positive and negative dependence can be easily captured by copula.

Consequently, copula has emerged as a very popular method for modelling dependence among assets and markets. Copulas are functions that connect multivariate distributions to their one-dimensional margins (Sklar, 1959). According to Nelsen (2006), copula can be seen from two points of view: "From one point of view, copulas are functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions. Alternatively, copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval (0,1)".

Mendes (2005) study asymmetry between the markets using copulas. Jondeau and Rockinger (2006) use copula to model dependence between the daily returns of four indices, namely, S&P500, FTSE, DAX, and CAC. Xu and Li (2009) estimate tail dependence across three Asian futures markets using three Archimedian copula families. Nikoloulopoulos, Joe and Li (2012) study relationships between equity indices using copula. Bartram et al. (2007) use copula to analyze the effect of introducing the Euro on the dependence between seventeen European stock markets during the period 1994-2003. Okimoto (2008) use regime-switching copulas to model dependence between pairs of US-UK and other G7 countries. Rodriguez (2007) utilize the copula model to investigate contagion.

It has been well-established that stock markets tend to crash together but not boom together the dependence structure should be examined on both tails of the return distribution (Patton, 2012). Consequently, we need a copula to model this asymmetry in the dependence structure, particularly at the upper and lower tail dependence coefficients (McNeil et al., 2005).

Modelling dependence relying on the available information leads to study of conditional copula. Modelling unconditional or time-varying copula requires specifying models for the unconditional or conditional marginal distributions, respectively, of the standardized residuals. As stated by Patton (2006), modelling the dependence structure of the variables directly using the unconditional probability yields a model for the unconditional copula of the returns.

In this paper, we therefore employ the Gumbel, Rotated Gumbel, Clayton, Rotated Clayton, Frank, Student-*t* and symmetrized Joe–Clayton (SJC) copulas for our analysis and used the Gaussian copula as the benchmark. It should be noted that the Frank copula implies asymptotic tail independence because it is symmetric assigning zero probability to events that are deep in the tails. Clayton and Gumbel copulas imply dependence in one of the tails, but not in the other. Specifically, Clayton copula assigns more probability mass to events in the left tail markets crash together) while Gumbel assigns more probability mass to events in the right tail (markets boom together). The two are therefore used to capture the right and left tail dependences, respectively. The SJC copula allows the tail dependences to be either symmetric or asymmetric.

A clear introduction to copula in dependence modelling is given by Nelsen (2006) and Joe (1997). A detailed treatment of copula with emphasis on risk management is given by McNeil, et al. (2005). Patton (2009) present a summary of applications of copula to financial time series and an extensive list of references. Patton (2012) review the literature on copula-based models for economic and financial timeseries.

3.0 Methodology

3.1 The model for the marginal distribution

Generalised Autoregressive Conditional Heteroscedasticity (GARCH) methodology is often used to model heteroscedasticity in fnancial series, where ARMA (p,q) models the conditional mean and GARCH (p,q) describes the conditional variances. The p,q refer to the autoregressive and moving average part, respectively, in the ARMA model. For the GARCH model, the order of GARCH is p and q is the order of the ARCH part. However, we have established in the previous section that the 'sylised facts' of equity returns experience joint negative extremes more often than joint positive extremes. The well-known Glosten, Jagannathan, and Runkle (GJR) 1993 model is ideal for modelling this behavior. Therefore given the presence of asymmetry in stock market returns and as modelled by Wang et al (2011) and Patton (2012), we employ GJR model to describe the marginals in this paper. The GJR(*p,q*) conditional variance model includes *p* past conditional variances composing the GARCH polynomial, and *q* past squared innovations for the ARCH and leverage polynomials.

Glosten, Jagananthan and Runkle (1993) develop the GARCH model which allows the conditional variance to have a different response to past negative and positive innovations. GJR GARCH captures the propensity for the volatility to rise more subsequent to large negative shocks than to large positive shocks, known as the "leverage effect".

A GJR-GARCH(P,O,Q) process for conditional variance h_t is defined as

$$r_t = u_t + \varepsilon_t \tag{1}$$

$$h_t = \omega + \sum_{p=1}^{P} \alpha_p \varepsilon_{t-p}^2 + \sum_{o=1}^{O} \gamma_o \varepsilon_{t-o}^2 I_{[\varepsilon t-o<0]} + \sum_{q=1}^{Q} \beta_q h_{t-q} \tag{2}$$

$$\varepsilon_t = h_t e_t \tag{3}$$

where e_t is normally distributed, μ_t can be any adapted model for the conditional mean and $I[\varepsilon_{t-o}<0]$ is an indicator function that takes the value 1 if $\varepsilon_{t-o} < 0$ and 0 otherwise. The conditional variance equation contains the ARCH terms ε_{t-p}^2, the GARCH terms h_{t-p} and the GJR terms $\varepsilon_{t-o}^2 I_{[\varepsilon t-o<0]}$. Other distributions for the error term include student t, skewed student and Generalized Error Distributions, and may involve more parameters as well as restrictions. The error term e_t of the mean equation (1) and is assumed to have a time-varying conditional variance h_t as specified in (2).

3.2 Copulas and their Dependence Concept

Copulas are functions that couple or link marginal distributions into their joint multivariate distribution. Sklar's theorem (1959) states the relationship between copulas and multivariate distribution functions.

By Sklar's (1959) theorem proves that each multivariate distribution with continuous marginals has a unique copula representation so that any function C: $[0,1]^n \rightarrow [0,1]$ satisfying some regularity restrictions implies a copula.

The bivariate version of this theorem, which is of interest in this study, is as follows: Let *H* be joint distribution function with margins *F* and *G*, then there exists a copula *C* that, for all x, *y* in $[0,1]^2$,

$$H(x, y) = C(F(x), G(y))$$

For continuously-distributed *F* and *G*, then *C* is unique; otherwise *C* is uniquely determined only on the range of *F* and *G*. Conversely, for copula *C* is and distribution functions *F* and *G* then *H*, is a joint distribution function with margins *F* and *G*.

Sklar's theorem makes it possible to separate a continuous multivariate distribution functions into the univariate margins and the multivariate dependence structure where the dependence structure is represented by the copula.

We begin with a brief description of Archimedean copulas and their properties as stated in Nelson (1999). The Archimedean copula is relative simple, covers many common probability distributions, and therefore used in a wide range of applications.

The general expression of Archimedean copulas for variables $u, v \in [0, 1]^2$ is

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v))$$

in which φ is the generator of the copula. The generator φ is a strictly decreasing function, which maps the interval (0,1] onto $[0,\infty)$.

3.2.1 Gumbel copula

This copula belongs to the special class of Archimedean copulas and is characterized by its stronger dependence in the lower probabilities. The Gumbel copula is given by

$$C_\theta^{Gumbel} = (u, v) = \exp\left(-[(-\ln u)^\theta + (-\ln v)^\theta]^{-1/\theta}\right) \quad \theta \in [1, \infty)$$

This copula is characterized by lower tail dependence and upper tail independence. Its main properties are:

a) $\theta = 1$ implies the independent copula
$C(u, , v; 1) = uv$

b) As $\theta \to \infty$, $CCl(u,v;\delta) \to \min(u, v)$. This limit is the upper Frèchet-Hoeffding bound.

c)) Lower Tail Dependence: $\lambda_L = 0$

d) Upper Tail Dependence: $\lambda_U = 2-2^{-1/\delta}$

3.2.2 Clayton Copula

Clayton Copula is characterized by upper tail dependence and lower tail independence and is given by:
$C_{Cl}(u,v;\theta) = \{u^{-\theta} + v^{-\theta} - 1\}^{-1/\theta}$

3.2.3 Frank Copula

This copula is characterized by upper and lower tail independence and is given as:

$C_F(u,v;a) = (-1/a)\ln(1 + (e^{-au}-1)(e^{-av}-1)/(e^{-a}-1))$ $\quad a \in [1, \infty)$

3.2.4 Student t Copula

This copula is constructed from the Student bivariate distribution based on Sklar's theorem. It is a two-parameter copula with the flexibility that the marginal univariate distribution functions don't have to be Student t with the same degrees of freedom.

Let $\Theta = \{(\nu, \Sigma) : \nu \in (1, \infty), \Sigma \in \mathbb{R}^{m \times m}\}$ where $t_{\nu,\Sigma}$ is the multivariate Student's *t* distribution with a correlation matrix Σ with ν degrees of freedom.

The Student's *t* copula can be written as

$$C_{\Theta}(u_1, u_2, \ldots u_m) = t_{\nu,\Sigma}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2), \ldots, t_\nu^{-1}(u_m)\right)$$

3.2.5 Gaussian copula

The Gaussian copula is given as:

$$C(u, v \mid \rho) = \Phi\left[\Phi^{-1}(u), \Phi^{-1}(v) \mid \rho\right].$$

The Gaussian and the Student *t* copula are elliptical copulas and do not have a closed-form expressions, but their density functions are available. It should be noted that as its degree of freedom increases, the Student *t* copula converges to the Gaussian model. When the degree of freedom is limited, the student t copula substantially differs from the Gaussian copula. Gaussian copula is asymptotically independent in both tails, while the student t copula has both upper and lower tail dependence of the same magnitude as a result of its radial symmetry.

3.2.6 The Symmetrized Joe–Clayton copula

Joe (1997) constructs the copula by taking a particular Laplace transformation of Clayton's copula. The Joe–Clayton copula is:

$$C_{JC}\left(u, v \mid \tau^U, \tau^L\right) = 1 - \left(1 - \left\{\left[1 - (1-u)^k\right]^{-\gamma} + \left[1 - (1-v)^k\right]^{-\gamma} - 1\right\}^{-1/\gamma}\right)^{1/k} \quad 3,$$

where $k = \dfrac{1}{\log_2(2 - \tau^U)}$, $\gamma = \dfrac{-1}{\log_2(\tau^L)}$ and $\tau^i \in (0,1)$

The original JC copula has some minor asymmetry when the two tail dependence measures are equal. To solve this problem, Patton (2006) proposes the "Symmetrized Joe–Clayton" copula (SJC) which is a simple function of $C_{JC}(\cdot)$:

$$C_{SJC}\left(u, v \mid \tau^U, \tau^L\right) = \frac{1}{2}\left(C_{JC}\left(u, v \mid \tau^U, \tau^L\right) + C_{JC}\left(1-u, 1-v \mid \tau^U, \tau^L\right) + u + v - 1\right)$$

It should be noted that this paper uses copula, copula model and copula function interchangeably, all referring to the same concept.

3.3 Estimation of Copula Parameters

As stated by Fan and Patton (2014), Sklar's theorem has enabled a lot of flexibility in multivariate modeling because the marginal distributions and the copula need not belong to the same family of distributions. The univariate models can be symmetric or skewed, continuous or discrete, fat-tailed or thin-tailed while the joint cumulative density function could be any copula function C, coupling any univariate cumulative distribution function (CDF) F, and any univariate CDF G will be a valid CDF. (Fan and Patton, 2014).

There are two main steps of estimating multivariate models using copula. First, there is a two-step procedures for the identification and estimation of the joint CDF, where the identification and estimation of the marginals are carried out as the first step, and the second step is the identification and estimation of the copula function. This is also referred to as Inference for Margins (Dias, 2004). Second, one-stage maximum likelihood estimates are obtained by maximizing sum of copula likelihood functions for all observations in one step by plugging all the required parameters into the copula (Patton, 2006).

The two-stage maximum likelihood (ML) provides precise estimation of structure and dynamics of the variable of interest. Estimates of parameters in two stage approach are asymptotically as efficient as one-stage estimation (Patton, 2006). However, it is difficult to apply one-stage ML for large samples in practice, because the optimal solution is the one which maximizes the sum of *n* likelihood functions for all the parameters simultaneously. This is because all the parameters are simultaneously optimized both for the margins and the dependence structure. This method can lead to challenging numerical and computational problems as the number of variables increase. The complexity of the estimation procedure can be decreased by using the two-step procedure, the Inference Functions for Margins (IFM).

Copula modelling using IFM requires selecting a parametric family for each of the margins. There is a need to choose the ideal distribution for the margins so as to avoid marginal model risk.

3.4 Copula-Based Dependence Parameters

It is well-known that copulas are invariant under strictly increasing transformations of the random variables. Spearman's ρ and Kendall's τ share this characteristic and can therefore be expressed in terms of copulas.

Given a set where the data values x and y are organized in order of size. The rank correlation coefficients can then be computed for the given numerical values, which are in the form of ranks.

As stated by McNeil, et al. (2005), rank correlations are simple scalar measures of dependence that do not rely on the marginal distributions rather depend only on the copula of a bivariate distribution. linear correlation, however, depends on both. Rank correlation may be calculated by looking at the ranks of the data alone. There are two main types of rank correlation are Kendall's tau and Spearman's rho. Kendall's tau is defined as a measure of concordance for bivariate random vectors while Spearman's rho is simply the linear correlation of the probability-transformed random variables. A third copula-based measure of dependence, as defined by McNeil, et al. (2005), is coefficient of tail dependence which measures the strength of dependence in the tails of a bivariate distribution.

3.5 The Data

We consider the daily returns on the following indices: the Nigerian Stock Exchange All Share Index, Ghana GSE Composite Index, South Africa Johannesburg Stock Exchange All Share Index, Egyptioan Thomson Reuters Egypt Index, Argentina Buenos Aires SE Merval Index, Indonesian Jakarta SE Composite Index, Mexican S&P/Bmv Ipc, Turkish BIST All shares Index, Hong Kong S&P/HKEx LargeCap, United Kingdom FTSE 100 Index, United States S&P 500 Index and the United States Dow Jones Industrial Average Index. The sample covers 2758 observations from 4th January 2007 to 29th December, 2017.

4.0 Empirical Analysis

It should be noted that the sample period covers a few extreme events like the Global Financial Crisis (GFC), the Nigerian banking crisis in 2009 and the recent Nigerian exchange rate crisis. A time series plot of these series over the whole sample period is presented in Figures 1 and 2. All algorithms were developed using the MATLAB® software package.

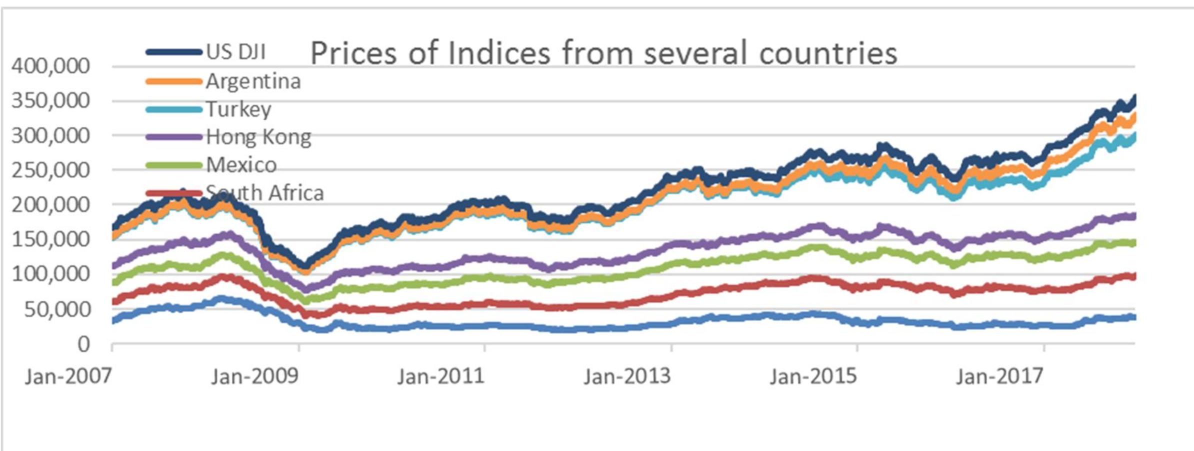


Figure 1: Prices of Indices from selected countries

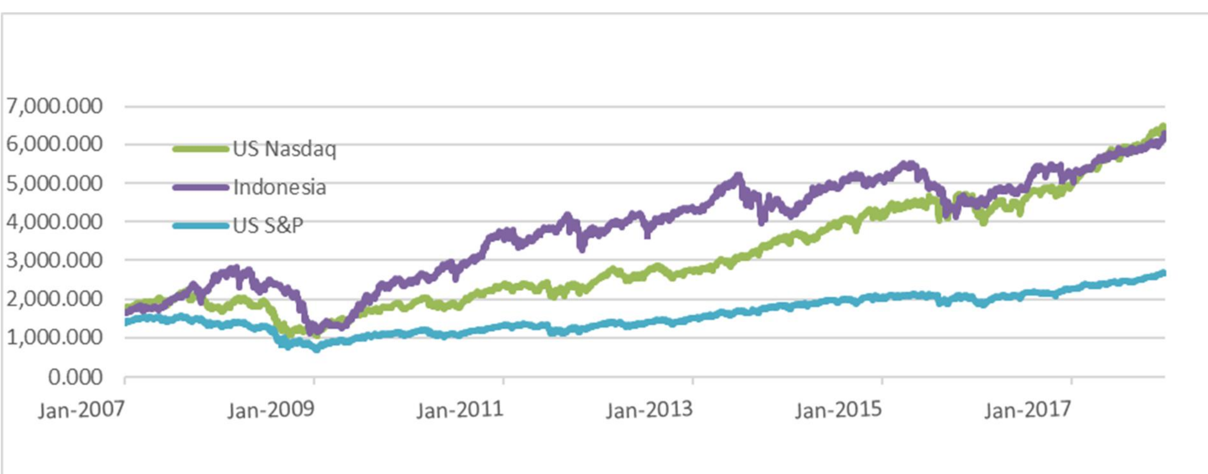


Figure 2: Prices of Indices from selected countries

Some of the log-returns for the series used in the analysis are plotted in figure 3.

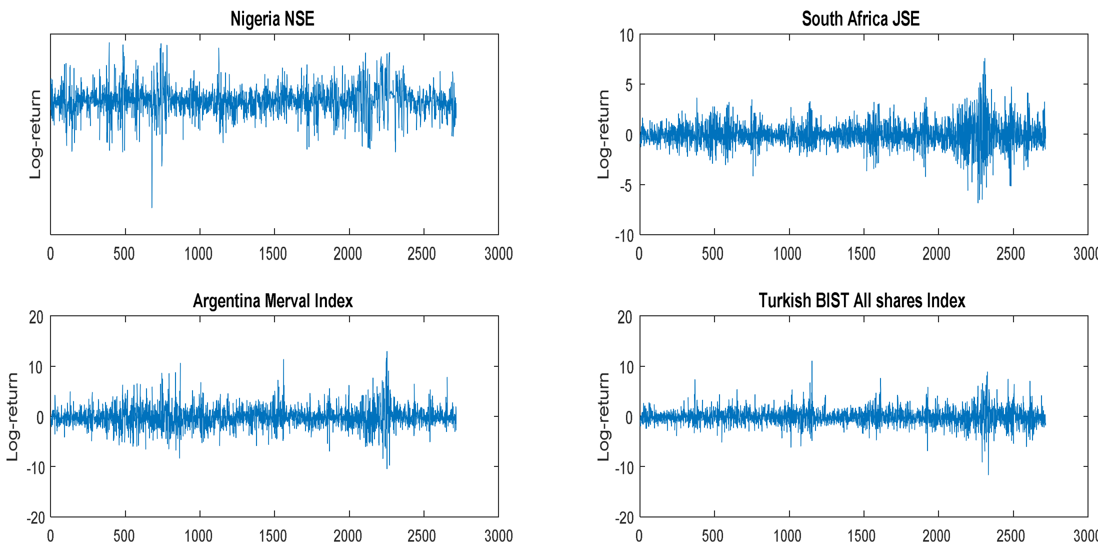
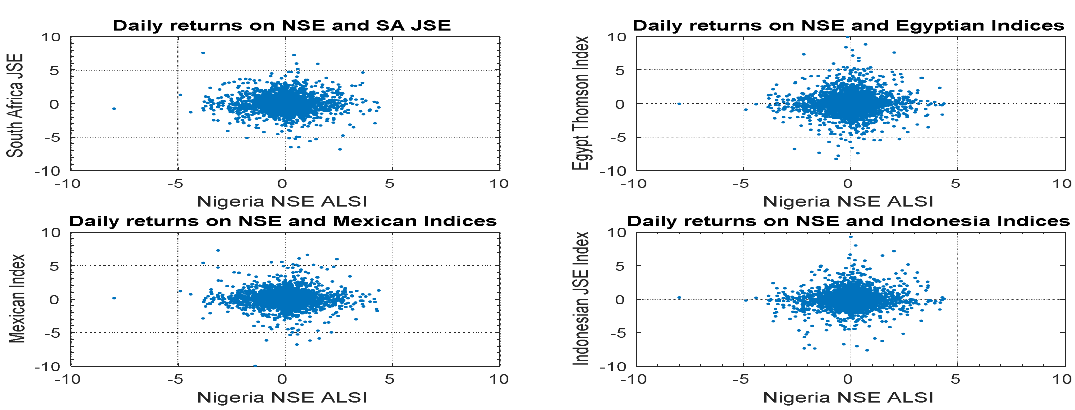


Figure 3: Returns of Nigeria NSE, South Africa JSE, Argentina Merval and Turkish BIST indices.

It is clear from figure 3 that the log-returns vary over time, but whether they are also interdependent is more difficult to tell. There are some quarters of the return series that seem to have higher variance than the others. This volatile behavior indicates conditional heteroscedasticity. Also, the series seems to fluctuate at a constant level. Therefore, there is a need to account for autocorrelation and heteroscedasticity in the marginal series.

To get a feeling for the dependence between X and Y, it is traditional to look at the scatter plot of the pairs $X_1,Y_1, \ldots ,X_n,Y_n$. Figure 4 presents a scatter plot of the stock indices returns.



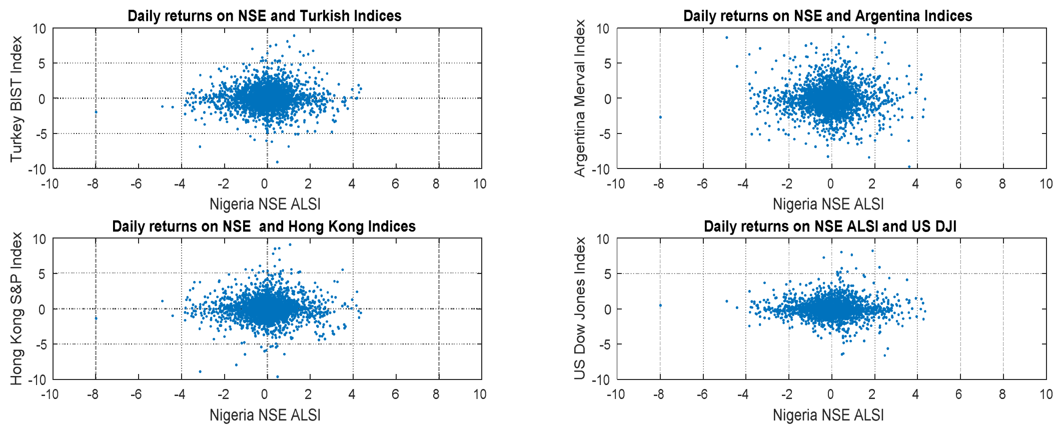


Figure 4: Scatterplots of the pairs

The summary statistics of the daily returns of the indices for the samples is presented in Tables 1. The summary includes the mean returns, the standard deviations, the skewness and the kurtosis for all the indices included in the sample.

Table 1: Summary statistics of the daily returns of the indices for the sample

	Mean	**Median**	**Std Dev**	**Skewness**	**Kurtosis**
Nigerian Stock Exchange ALSI	-0.0052	0.00033	1.0688	-0.1874	6.1029
Ghana GSE Composite Index	-0.0551	-0.0223	0.6782	0.0775	30.863
South Africa Stock Exchange ALSI	-0.0298	-0.0615	1.2210	0.1392	6.807
Egyptian Thomson Reuters Index	-0.0297	-0.1365	1.5264	1.4983	15.227
Argentina Merval Index	-0.1005	-0.1369	2.0278	0.5195	6.883
Indonesian Jakarta Composite Index	-0.0496	-0.1158	1.3539	0.6463	11.064
Mexican S&P/Bmv Ipc	-0.0213	-0.0410	1.2177	-0.1462	10.833
Turkish BIST All shares Index	-0.0386	-0.0822	1.5895	0.3258	7.691
United Kingdom FTSE 100 Index	-0.0069	-0.0347	1.2300	0.1259	10.637
US S&P 500 Index	-0.0243	-0.0592	1.2730	0.3419	13.82
US Dow Jones Industrial Index	-0.0263	-0.0551	1.1692	0.1038	13.694

Although the daily mean returns are all negative, they are all close to zero, except Merval Index of Argentina with slightly higher value than others, indicating that the average return is almost constant over time. For the sample, the median of the daily returns are all negative values, except Nigeria, implying that all the indices offer lower returns in comparison to Nigeria NSE.

The sample statistics also shows that none of the stock indices had a significant trend with the means being very small relative to their associated standard deviation. The minimum and maximum daily standard deviations of the sample are achieved by Ghana and Argentina indices, respectively.

The occurrence of negative skewness and excess kurtosis in stock market returns is in line with previous numerous empirical studies. The returns' skewness is mostly positive for the sample, except Nigerian and Mexican indices that reported negative values. From the values obtained for the kurtosis we can see that, for all the series in the sample, the unconditional univariate distributions of the returns are clearly heavy tailed suggesting non-normality.

Also, we performed the Jarque-Bera test on each of the log-returns of the indices with the null hypothesis that the data in each of the series come from a normal distribution with an unknown mean and variance. The tests rejected the null hypothesis at the 5% significance level for each of the return series thereby strongly rejecting normality. This feature has been found in the financial econometric literature when stock indices are modelled.

4.1 Models for the marginal series

The first step of the IFM method requires choosing a family of distributions to model each univariate return time series, independently of any copula model. Therefore, before modelling the dependence structure between the returns, we must first model the marginal return series of the individual stock indices over the 3 samples.

In this paper, the conditional mean of each time series will be examined using an ARMA type model and the conditional variance using a GARCH type model. We will also construct the estimated standardized residuals based on Normal or the Student t distributions.

Ljung-Box and ARCH tests on the residuals and their squared values are used to detect the presence of autocorrelation and heteroscedasticity, respectively, in sample series. At 5, 10, and 15 lags using Ljung-Box Q-test, we tested for residual autocorrelation with the null hypothesis that the residuals are not autocorrelated. We also tested the hypothesis that there are significant ARCH effects (conditional heteroscedasticity, a discrete-time version of stochastic volatility) in the residuals of the stock indices returns using 5, 10 and 15 lags.

Using the sample for each of the indices, the *p-values* of Ljung-Box tests suggested that there is significant autocorrelation in the residuals at the chosen levels. ARCH test of Engle (1982) for all series in the sample also indicated that there are significant conditional heteroscedasticity (ARCH effects) in the residuals of the returns for all indices in the sample. We concluded that the null hypothesis of homoscedasticity should be rejected for the returns of the relevant indices using the *p-values*. As observed from plots in Figure 3, the test confirms the series exhibit conditional heteroscedasticity, that is, large changes in the returns tend to cluster together just like small changes. Given that all the returns are leptokurtic, the the error term could be modelled using Student t distribution.

We therefore proceed to obtain the optimal ARMA GJR-GARCH model for the daily returns of the indices. First, we consider ARMA models for the conditional mean up to order (5, 5) and the optimal models were found using the Bayesian Information Criterion (BIC) as shown in Table 2.

Table 2: Conditional Models evaluated using BIC Criteria

	AR	**MA**
Nigerian Stock Exchange ALSI	1	0
South Africa Stock Exchange ALSI	0	0
Egyptian Thomson Reuters Index	1	0
Argentina Merval Index	0	0
Indonesian Jakarta Composite Index	1	0
Mexican S&P/Bmv Ipc	0	0
Turkish BIST All shares Index	0	0
United Kingdom FTSE 100 Index	0	0
US S&P 500 Index	0	1
US Dow Jones Industrial Index	0	1

Next, we consider volatility models in the GJR-GARCH class and select the optimal models ARMA (p,q) GJR-GARCH(1,1) as studied by Wang et al (2011). We therefore specified, and then fitted a ARMA(p,q)-GJR(1,1) model to the indices returns series. The fitted ARMA conditional mean models used in the estimation selected using the BIC Criteria are given in Table 2. As earlier stated, the ARMA(p,q) models compensate for autocorrelation, while the GJR-GARCH(1,1) models compensates for heteroskedasticity and also incorporate asymmetry into the variance equation.

The GJR-GARCH models were estimated with both Normal and Student *t* distributions. Table 3 reports the maximum likelihood estimates in the first row and standard errors of the parameters of the marginal distribution models for the indices based Student *t* distribution in the second row. The right-most column returns the value of the loglikelihood objective function for Student *t* distributions at the top and Normal distributions at the bottom.

Table 3: Estimated parameters for the GJR-GARCH(1,1)-ARMA(p,q)-t marginal distributions for the Sample

	Conditional Mean (only USS & USD were MA1)		**Conditional Vraiance**					
								Log Likelihood
	Mu	**AR1**	**Omega** **Ω**	**Alpha** **α**	**Gamma** **γ**	**Beta** **β**	**Shape**	
Nigerian	0.01568 0.01351	**0.40833** **0.01993**	**0.11217** **0.02905**	**0.28753** **0.04978**	-0.02991 0.04734	**0.63521** **0.05982**	**4.43593** **0.41910**	-3373.035 -3466.987
South Africa	**-0.095158** **0.016631**		**0.006067** **0.003031**	**0.042534** **0.012017**	**1.000000** **0.265212**	**0.925247** **0.007229**	**10.0000** **1.298251**	-3872.101 -3874.125
Egyptian	**-0.13697** **0.01952**	**0.21379** **0.01914**	**0.07463** **0.01738**	**0.19407** **0.02554**	**0.34447** **0.05095**	**0.78840** **0.02405**	**5.61779** **0.61137**	-4367.624 -4441.306
Argentina	**-0.18945** **0.02982**		**0.14349** **0.04113**	**0.12747** **0.02284**	**0.32592** **0.06387**	**0.83982** **0.02660**	**5.80273** **0.69206**	-5399.425 -5454.857
Indonesian	**-0.110803** **0.016736**	**0.039945** **0.019325**	**0.027938** **0.007486**	**0.110131** **0.019948**	**0.521562** **0.090795**	**0.867039** **0.018112**	**5.672716** **0.638362**	-4019.481 -4086.859
Mexican	**-0.059838** **0.015590**		**0.005968** **0.002368**	0.031606 0.017915	1.000000 0.572237	0.938941 0.006019	7.855647 1.167006	-3753.509 -3788.487
Turkish	**-0.12311** **0.02433**		**0.05377** **0.02045**	**0.08559** **0.01878**	**0.29916** **0.08088**	**0.89256** **0.02335**	**6.61719** **0.84869**	-4772.829 -4820.94
UK	**-0.066384** **0.015343**		**0.008139** **0.002829**	**0.045792** **0.014357**	**1.00000** **0.29818**	**0.913410** **0.008171**	**8.245244** **1.290237**	-3754.832 -3783.874
US S&P 500	**-0.102038** **0.012254**	**-** **0.075124** **0.019285**	0.004061 0.002103	0.060259	1.00000	**0.903349** **0.008772**	**6.225520** **0.826284**	-3554.706 -3596.672
US Dow Jones	**-0.100542** **0.012044**	**-** **0.059935** **0.018893**	**0.005185** **0.002354**	**0.091726** **0.018640**	**0.592439** **0.113943**	**0.895822** **0.013287**	**6.165948** **0.793943**	-3390.441 -3434.692

Bolded estimates denote significance at 5%

The residual series of all index returns for each sample passed the goodness of-fit test using the Kolmogorov–Smirnov test and therefore could not reject the null hypothesis that the
t model is well-specified for the return series. From the table, the estimated degrees of freedom are relatively small (between 6 to 8), indicating significant departure from normality. This has been further collaborated by the estimated log likelihood that favour using Student's t distribution instead of Normal for all indices.

This implies that the selected GJR-GARCH(1,1)-ARMA(p,q)-t marginal distributions models for each index are the right choice and we can proceed with modelling the dependence structure through examination of summary statistics and using copula functions.

4.2 Estimating dependence using summary statistics

The most popular measure of dependence is Pearson/linear correlation. Pearson/linear correlation is a simple measure dependence that should satisfy two conditions before it can be relied upon. The data in the pairs must be normally distributed and should be in the same frequency. Recall that we have already discussed the limitations of linear correlation as a dependence measure, particularly multivariate non-normal and non-elliptical distributions. Alternative dependence measures based on copulas, the rank correlations have also been argued. Rank correlations are useful for calibration of copula to data (McNeil et al., 2015).

Table 4 shows the Pearson, Spearman, and Kendall correlations between Nigeria's index return and the other index returns.

Table 4: Summary statistics of the stock market returns

Nigerian ALSI vs	**Pearson**	**Spearman Rho**	**Kendall Tau**
South Africa ALSI	0.003	0.013	0.009
Egyptian Index	-0.008	-0.002	-0.001
Argentinian Index	-0.005	0.004	0.003
Indonesian JSE Index	0.086	0.066	0.044
Mexican S&P	-0.031	-0.019	-0.013
Turkish BIST ALSI	0.028	0.019	0.013
UK FTSE 100 Index	0.025	0.035	0.024
US S&P 500 Index	-0.019	-0.026	-0.018
US DJI Index	-0.017	-0.025	-0.017

The following analysis is between the Nigeria NSE against the remaining nine indices.

From the table, the NSE has positive linear/Pearson with four markets while four markets indicated positive association based on Kendall's and Spearman's correlations. The highest positive Pearson, Kendall's and Spearman's correlations is with Indonesian JSE indices. The highest negative Pearson, Kendall's and Spearman's correlations is with Mexican, US S&P 500 and US S&P 500 indices, respectively.

For a given pair of association between Nigeria NSE and each country index return series, Pearson measure reported higher co-movement values in 5 out of 9 situations than Spearman's rho which also reports higher corresponding correlations than Kendall's tau.

Furthermore, the unconditional correlations between Nigeria NSE and other indices are small. The linear unconditional correlations using the sample range from 0.3% to -3.1%. The unconditional Spearman's rho ranges from 0.2% to -2.6% for the sample. 0.9% to -1.8% is the range of Kendall's tau estimates for the sample.

We should recall from Table 1 and relevant tests showed that the data is leptokurtic, therefore invalidating the use of linear correlation as a measure of dependence.

We will now consider a variety of parametric models for the copula of the indices return series.

4.3 Modelling dependence using Unconditional or Constant Copula

In this section, we consider copula functions with constant dependence parameter. After specifying models for each of the individual series in Section 4.2, we transform the observations into the unit square applying the probability-integral transformation. We then proceed to the next stage of the IFM method fitting copula families to the pseudo-observations.

In this paper, we therefore employ the Gumbel, Rotated Gumbel, Clayton, Rotated Clayton, Frank, Student-*t* and SJC copulas for our analysis and used the Gaussian copula as the benchmark. As earlier stated, the choice of copula models is to ensure that we have considered the dependence structure from theoretical and empirical literature that range from zero in case of asymptotic independence, to one in case of perfect asymptotic tail dependence, asymmetry as well as upper and lower tail dependence.

The goodness-of-fit is evaluated using loglikelihood criterion in line with Patton (2006) and Patton (2012). Table 5 presents the estimated parameters of the 8 different models for the copula of the standardized residuals of the indices. The value of the estimated copula parameter is given under the copula family and the next value, the loglikelihood, is given below the parameter estimated.

Table 5: Estimated copula parameter estimates and loglikelihood values

Nigerian ALSI vs								
	Gaussian	Student-t	Gumbel	Rotated Gumbel	Clayton	Rotated Clayton	SJC	Frank
South Africa ALSI	0.0072 -0.0703	0.0121, 0.123 -16.492	1.100 22.418	1.1000 15.3572	0.0404 -2.1465	0.0050 -0.0325	0.0000,0.1346 -1.7876	0.0793 - 0.2278
Egyptian Thomson Reuters Index	-0.0068 -0.0619	-0.004;13.53 -5.795	1.100 28.178	1.1000 29.307	0.0078 -0.0793	0.00026 -0.0101	0.052,0.105 1.6037	0.0017 0.0011
Argentina Merval Index	-0.002 -0.0056	0.0025, 9.153 -16.2497	1.100 26.9366	1.100 18.9600	0.0385 -1.9809	0.00014 0.0015	0.00,0.000 -1.3455	0.02822 - 0.0277
Indonesian Jakarta Index	0.0738 -7.4057	0.0739,16.069 -11.704	1.1000 -0.8544	1.1000 -1.5925	0.0811 -7.569	0.0747 -6.383	0.0012,0.0029 -10.1992	0.4164 - 6.0859
Mexican S&P/Bmv Ipc	-0.0253 -0.8703	- 0.0241,16.21 -5.5555	1.1000 41.9296	1.1000 32.8248	0.0157 -0.3303	0.0001 0.0104	0.1498, 0.028 2.3953	0.00122 0.0011
Turkish BIST ALSI	0.0224 -0.6827	0.0216,28.889 -2.0441	1.1000 22.7517	1.1000 22.3642	0.0323 -1.3133	0.0197 - 0.4941	0.0004,0.2968 -1.1850	0.1213 - 0.5203
United Kingdom FTSE 100 Index	0.0318 -1.3693	0.0361,9.4019 -14.4908	1.1000 14.5607	1.1000 9.6969	0.0585 -4.2478	0.0279 - 0.9160	0.0000,0.0013 -3.7602	0.2260 - 1.7625
US S&P 500 Index	-0.0304 -1.2540	- 0.0302,10.02 -12.4399	1.1000 42.3821	1.1000 27.1092	0.0340 -1.6268	1.08e-04 0.0154	0.000, 0.203 1.6557	0.0001 0.0012
US Dow Jones Industrial Index	-0.0271 -0.9995	- 0.0276,9.958	1.1000 40.8950	1.1000 26.9890	0.0353 -1.7533	0.0001 0.0130	0.0000, 0.201 1.3503	0.0001 0.0011

		-12.3474						

In Table 5, student t and SJC copula have two values in the first row. For student t, the first of the two values is the estimated parameter and the second is the degrees of freedom. It should be noted that the Gaussian copula can be derived as the limit of the student t copula when the degrees of freedom go to infinity.

Table 5 shows that all the copula functions for pairs of Nigerian stock market with those of South Africa, Indonesia, Turkey and United Kingdom have positive parameters. This indicates that Nigeria's ALSI return correlates positively with the indices returns of the other markets in the sample. In terms of copula functions for the pairs of Nigerian stock market and those of Egypt, Mexico and the 2 United States indices, Gaussian/Normal and Student t reported negative dependence parameters while other functions gave positive values. All other copula functions reported positive values for the pair of Nigerian stock market with that of Argentina, except the Gaussian/Normal copula function that reported negative dependence parameter.

The clayton copula function reported positive association between Nigerian ALSI and other stock markets in the following declining order: Indonesia, UK FTSE 100, South Africa, Argentina, US Dow Jones, US S&P 500, Turkey, Mexico and Egypt. Similarly, the rotated clayton copula gave strictly positive association between Nigerian ALSI and other stock markets in the following decreasing order: Indonesia, UK FTSE 100, Turkey, South Africa, Egypt, Argentina, US S&P 500 and jointly Mexico & US Dow Jones.

Though distinct from the other copula functions, the SJC copula function also gave positive association between Nigerian ALSI and other stock markets in the following declining order: Mexico, Egypt, Indonesia, Turkey, South Africa, UK FTSE, US Dow Jones, US S&P 500 and Argentina.

Frank copula function also produced positive association between Nigerian ALSI and other stock markets in the following declining order: Indonesia, UK FTSE 100, Turkey, Egypt, South Africa, Argentina, Mexico, US S&P 500 and US Dow Jones.

Both Gumbel and Rotated Gumbel gave 1.100 as the dependence parameter between pairs of Nigerian ALSI and each stock market.

Which copula function is the right one for the sample used in this study? The best or optimal copula based on the log-likelihood criterion is that with the lowest likelihood value. That is because the optimization process minimises the negative log-likelihood function in order to estimate the copula values.

From the functions fitted to the pseudo observations as shown in Table 5, we obtained that the Student t copula fits the data very well and therefore has emerged as the model to describe pair wise dependence between Nigerian ALSI against nine other world's stock markets. This finding is consistent with recent studies showing that the Student t copula often provides a much better fit of multivariate financial return data than the other unconditional copula models (Dias and Embrechts, 2010).

According to the log-likelihood criterion, the Clayton and SJC copulas emerged as the second and third best models, respectively, for describing Nigeria's pairwise dependence with other markets. Similarly, the worst copula functions to model the dependence is Gumbel followed by Rotated Gumbel copula.

Despite the above explanation, which stock market is highly integrated with Nigerian ALSI according to copula functions?

Tables 4 and 5 show the integration of Nigerian ALSI vs the other markets based on student t copula, linear and rank correlations. From the tables, as expected, the student t copula paramater, Kendall's tau and Spearman's rau, produced the same ranking of integration among the markets. The highest positive association, in decreasing order is with Indonesia, UK FTSE 100, Turkey, South Africa and Argentina. The same copula model showed negative dependence in increasing direction: Egypt, Mexico, US Dow Jones and US S&P 500. Linear correlation produced a different ranking as well as sign (positive and negative) association for some markets.

The exception is the Indonesian stock market that is the most highly integrated index with Nigeria, irrespective of the model or correlation coefficient. In Africa, the Nigeria market is more highly integrated with the South African over Egyptian market. In the developed nations, the Nigeria market is highly positively integrated with UK FTSE followed by lower but negative integration with the US markets.

Conclusion & Discussion

In this paper, we examined the relationship between Nigeria stock exchange and other markets. We performed a range of goodness-of-fit tests to help us select the best copula as a measure of dependence between Nigeria and each of the selected stock markets. We employed the Gumbel, Rotated Gumbel, Clayton, Rotated Clayton, Frank, Student-*t* and symmetrized Joe-Clayton copulas for our analysis and used the Gaussian copula as the benchmark.

The result showed that the Student t copula fits the data very well and therefore has emerged as the model to describe pair wise dependence between Nigerian stock index against nine other world's stock markets. This finding is consistent with recent studies showing that the Student copula often provides a much better fit of multivariate financial return data than the other unconditional copula models (Dias and Embrechts, 2010) in general and for stock market returns in particular.

The study revealed that, as expected, the student t copula parameter, Kendall's tau and Spearman's rho, produced the same ranking of integration among the markets. The highest positive association, in decreasing order is with Indonesia, UK FTSE 100, Turkey, South Africa and Argentina. The same copula model showed negative dependence in increasing direction: Egypt, Mexico, US Dow Jones and US S&P 500. Linear correlation produced a different ranking as well as sign (positive and negative) association for some markets. The exception is the Indonesian stock market that is the most highly integrated index with Nigeria, irrespective of the model or correlation coefficient. In Africa, the Nigeria market is more highly integrated with the South African over Egyptian market. In the developed nations, the Nigeria market is highly positively integrated with UK FTSE followed by lower but negative integration with the US markets.

The study showed that linear correlation is a wrong measure of association for stock returns between Nigeria's and several other indices. This has important implications for portfolio analysis by market players, the choice of which stock market the NSE selects for a deepened relationship as well as for contagion analysis.

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